Circuit analysis

What is a circuit?

- An electric circuit is the path through which electric current flows.
- For current to flow there must be a potential difference between the ends of the circuit (voltage difference)

Circuit analysis is the …

- Calculation of current **through** components (I, measured in amperes, A)

- Voltage drop **across** components (V, measured in volts, V)

  And, as a result

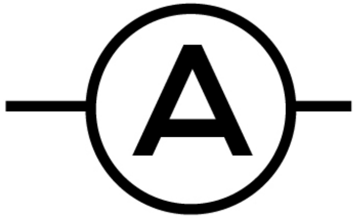
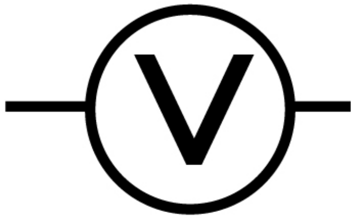
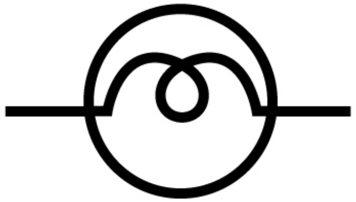
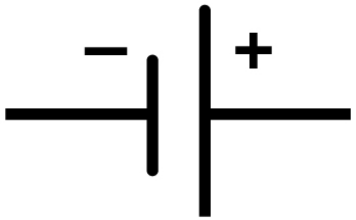
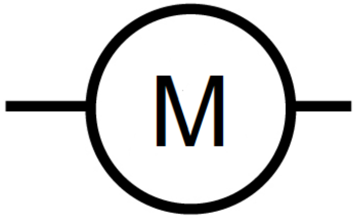
- The power available to a component (P = IV, measured in watts, W)

A component

- A component is any part of a circuit
  - Wires
  - Switch
  - Battery
  - Motor
  - Resistor
  - etc.

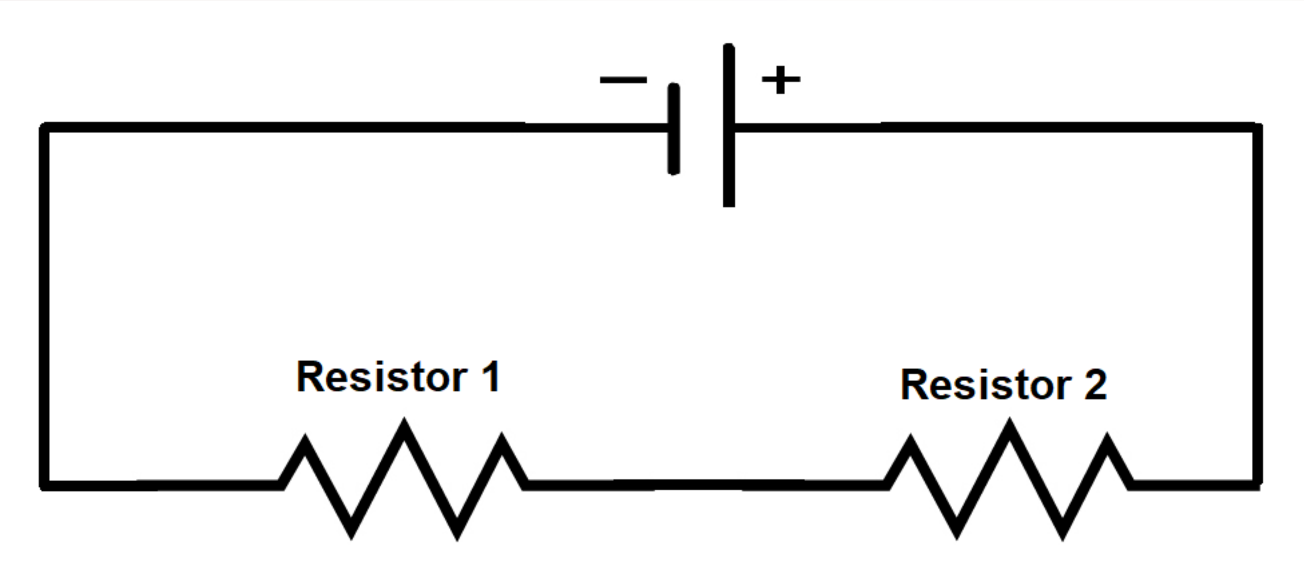
Circuit components

image courtesy BC Hydro

Ammeter	Voltmeter	Light bulb
		
Connection point	**Resistor**	**On/off switch**
		
Wire	**Battery**	**Motor**
		

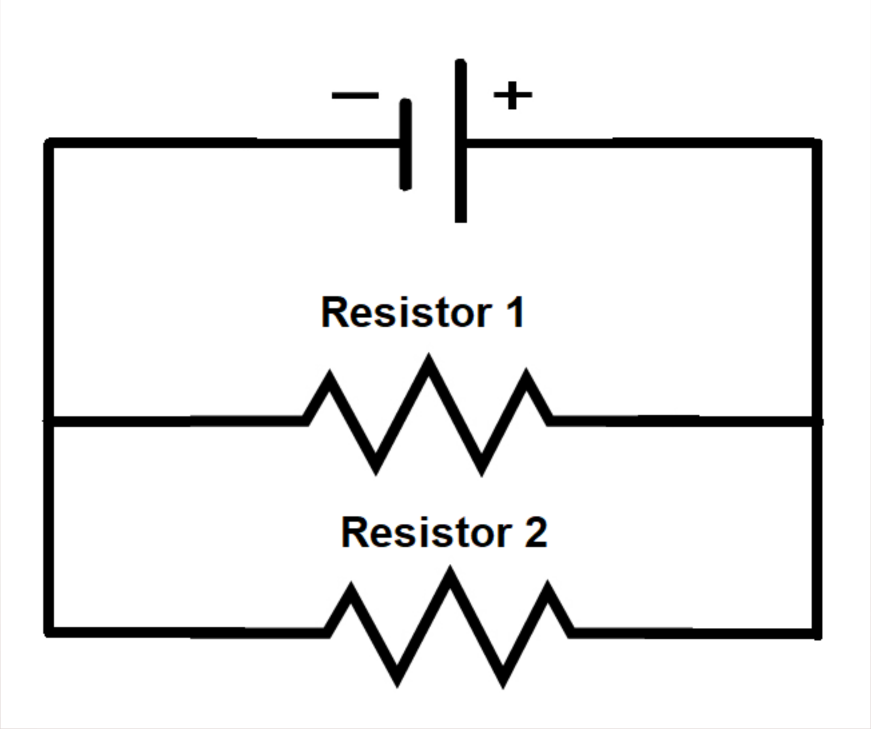
Terminology 1

- Components are in **SERIES** when they are on the same path in the circuit. Resistors 1 and 2 are in series below.



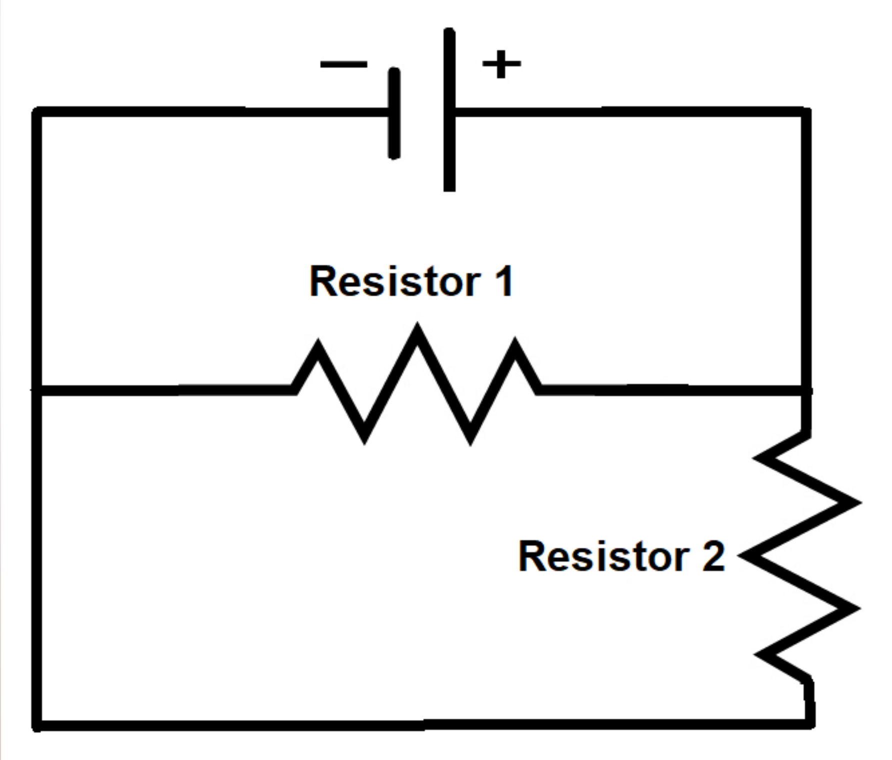
Terminology 2

- Components are in **PARALLEL** when they are on parallel paths in the circuit. Resistors 1 and 2 are in parallel below.



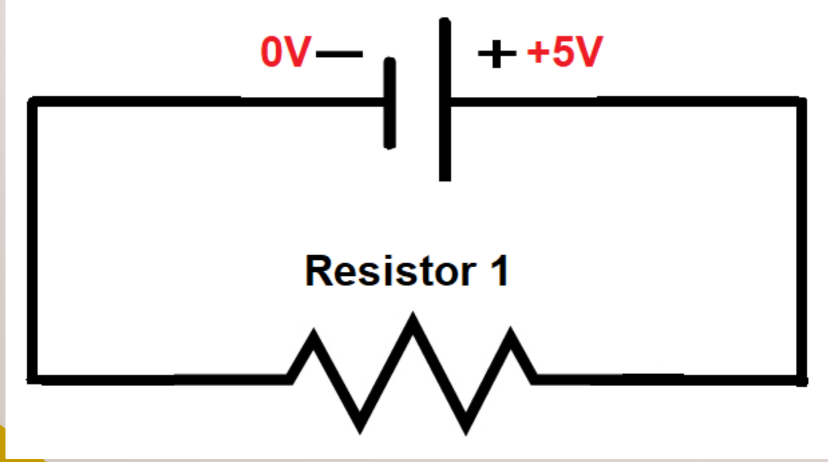
Warning

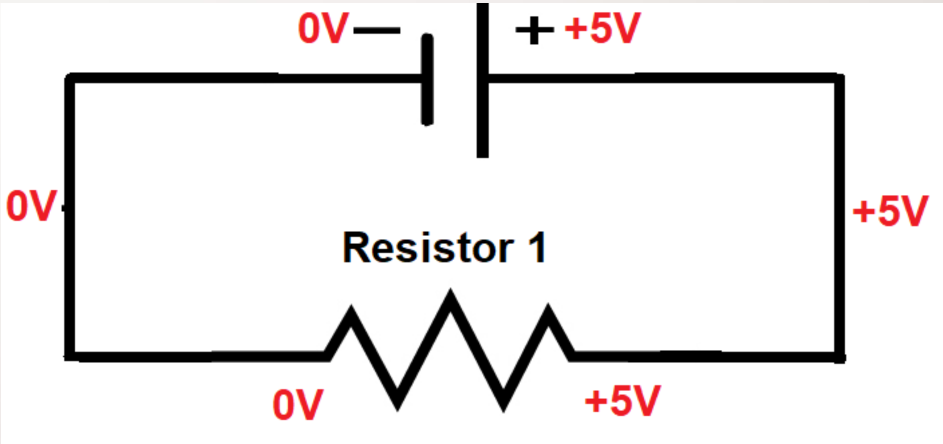
- “parallel” isn’t really parallel, just down separate paths, for example, resistors 1 and 2 are “parallel” in the diagram below.



A simple beginning

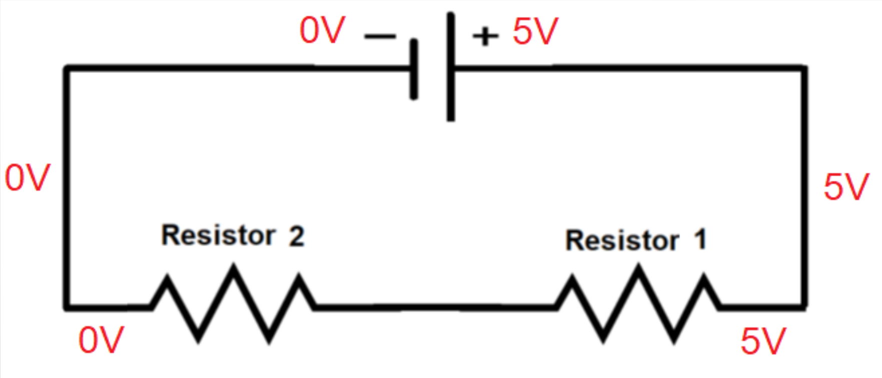
- Consider the circuit on the left, it has a 5V battery. If the right side of the battery has a potential of +5V, then that whole piece of wire has a potential of +5V and also, the whole left piece of wire has a potential of 0V
- Thus, the potential drop across the resistor must be 5V





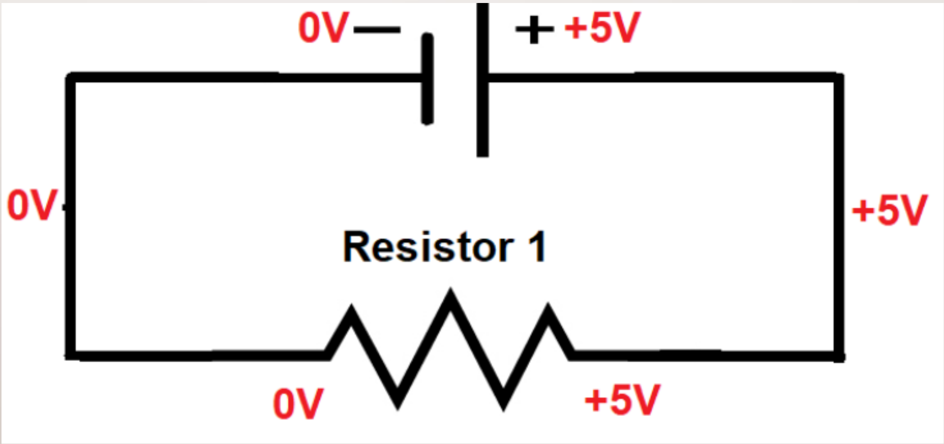
But … what if two resistors are in series?

- As seen below, the right wire has a potential of +5V, the left wire a potential of 0V. If the resistors are of equal resistance, then the potential drop across each must also be equal. So the wire in the centre must have a potential of 2.5V and the **circuit potential** drops 2.5V with each resistor.



But, lets back the truck up for a second …

- Below is a circuit where the voltage V drops across a resistance R.
- Note, it doesn't matter how big (or small) R is, the whole voltage will drop across it.
- If the resistance is big, the current will be slow
- And .. If the resistance is small, the current will be fast
- Ohm's Law gives us the relationship: $\boldsymbol{V = IR}$



Ohm's Law

$$V = IR$$

Or

$$I = \frac{V}{R}$$

- The amount of current that flows through a component(s) is equal to the size of the voltage across the component(s) divided by the resistance of the component

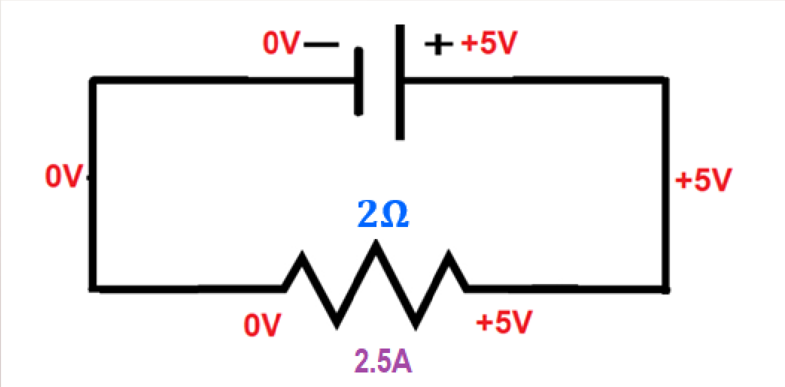
Quick calculation

- If the resistance in the resistor in the circuit below is 2Ω ("2 Ohms") then using Ohm's Law

$$V = IR \text{ or } I = \frac{V}{R}$$

- We can calculate that the flow of current (I) is

$$I = \frac{V}{R} = \frac{5}{2} = 2.5A$$



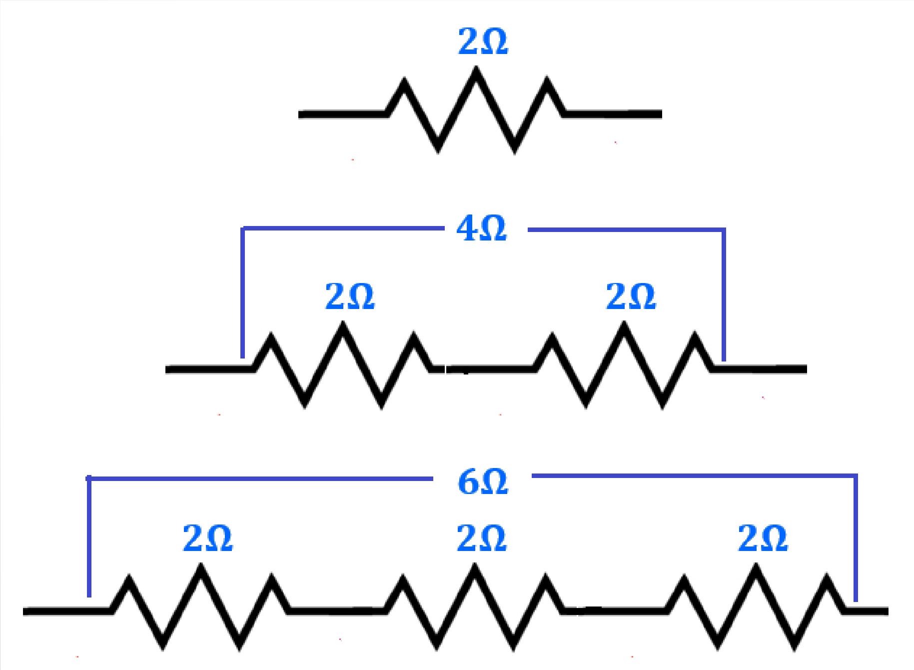
Ohm's Law …

- Ohm's Law works for any point(s) in a circuit at any point in time but …
  - Resistance can change as a result of voltage (potential) across component(s)
  - Or resistance can change with the temperature of components
  - Or resistance can change over time due to a change in the molecular

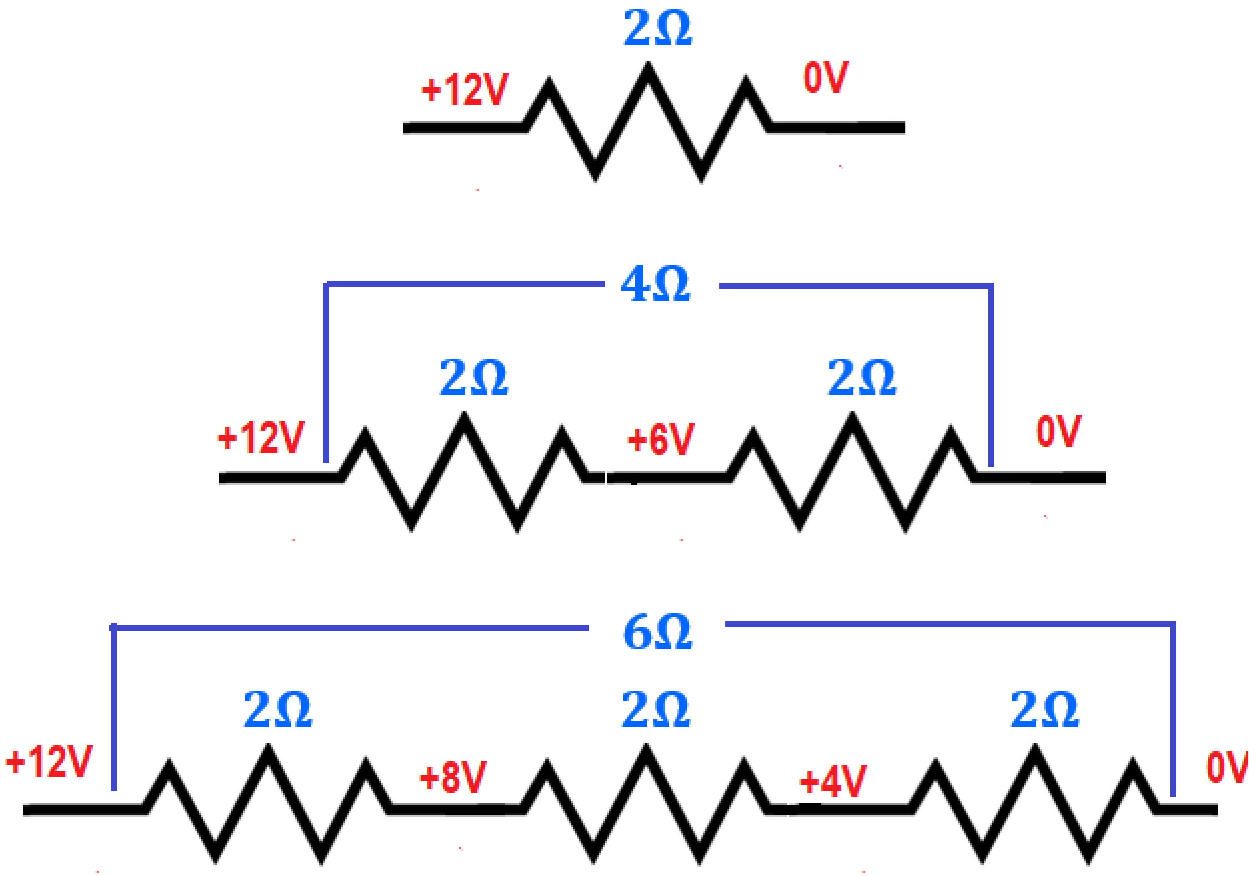
Add resistors in series

- What happens if we place resistors in series?
- The total resistance is simply the sum of the resistors

$$R_{total} = R_1 + R_2 + R_3 + \ \ldots$$



And Ohm's Law can be used on each ..



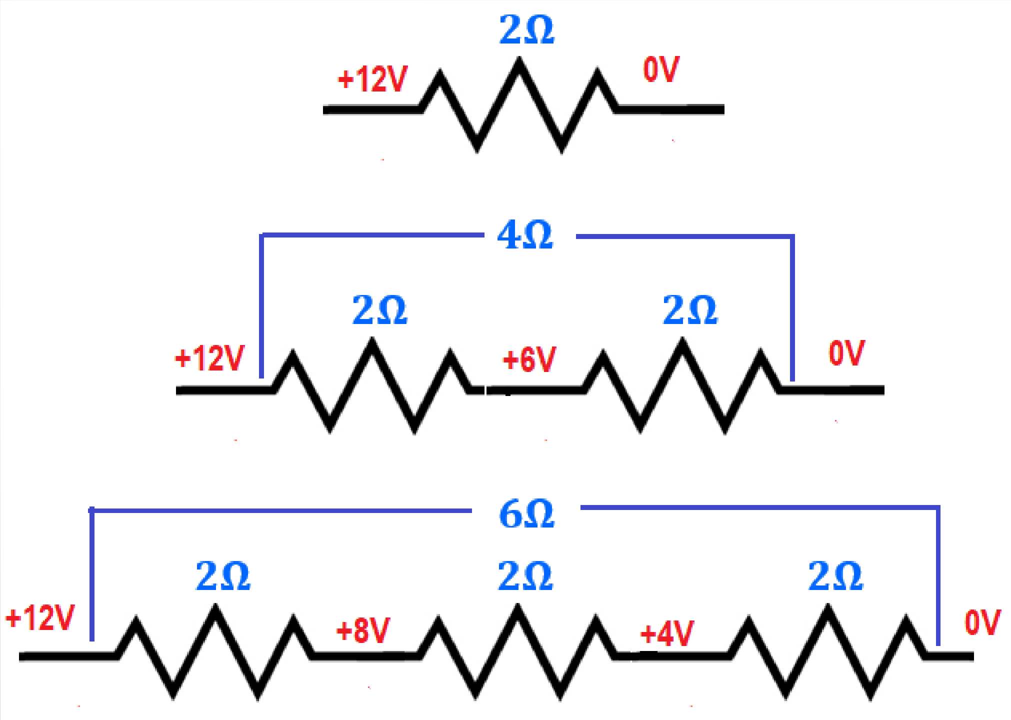
$$I = \frac{V_{total}}{R_{total}} = \frac{12}{2} = 6A$$

$$I = \frac{V_{total}}{R_{total}} = \frac{12}{4} = 3A$$

$$I = \frac{V_{total}}{R_{total}} = \frac{12}{6} = 2A$$

IFF (if and only if)

- The resistors are all the same resistance, then the voltage (potential) will drop equally across each resistor



But, fear not if the resistors are different

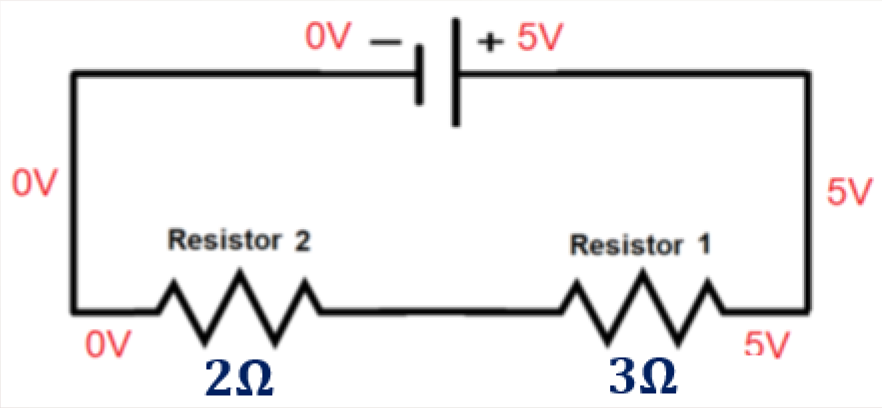
- Total resistance is still the sum of the resistances
- Thus current can still be calculated with Ohm's Law

$$(I_{total} = \frac{V_{total}}{R_{total}})$$

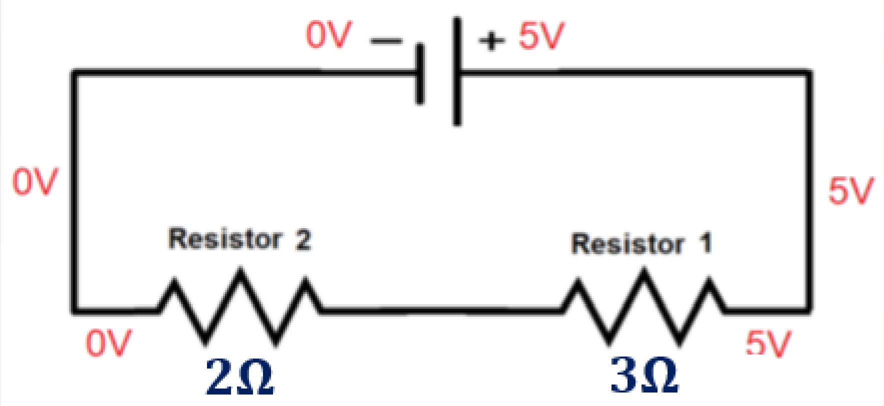
- NOTE: Current will be the same at all points along **that** section of wire, so I_{total} $is\ I$ for that section of wire.

And ….

- Once you have the current through the set of components
- You also have the current through each component
- Lets consider the example



Example



$$R_1 = 3\Omega, \qquad R_2 = 2\Omega, \qquad V_t = 5V$$

$$R_{t=} R_1 + R_2 = 2 + 3 = 5\Omega$$

$$V = IR \ \therefore I = \frac{V}{R} = \frac{5}{5} = 1A$$

Now we can use Ohm's Law again to calculate the voltage drop in each resistor

$$V_1 = IR_1 = 1 \times 3 = 3V \quad and\ V_2 = IR_2 = 1 \times 2 = 2V$$

Series recap

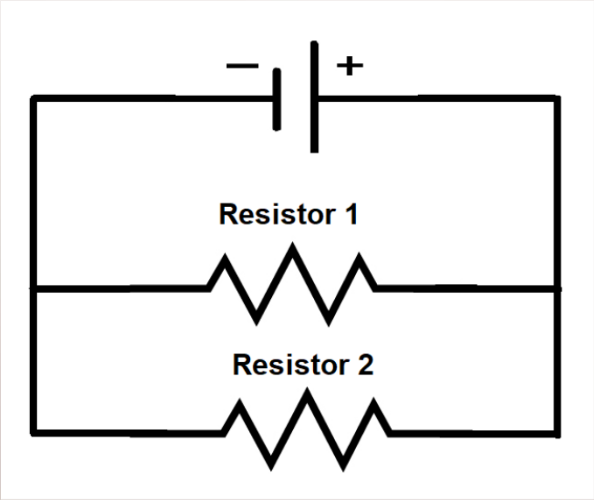
- Components on the same line are in series
- Total resistance on a series is the sum of resistances

$$R = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + \ \dots + \ R_n$$

- The current through a series is constant
- Ohm's Law can be used on individual components or on part or on the whole series

Parallel circuit lines

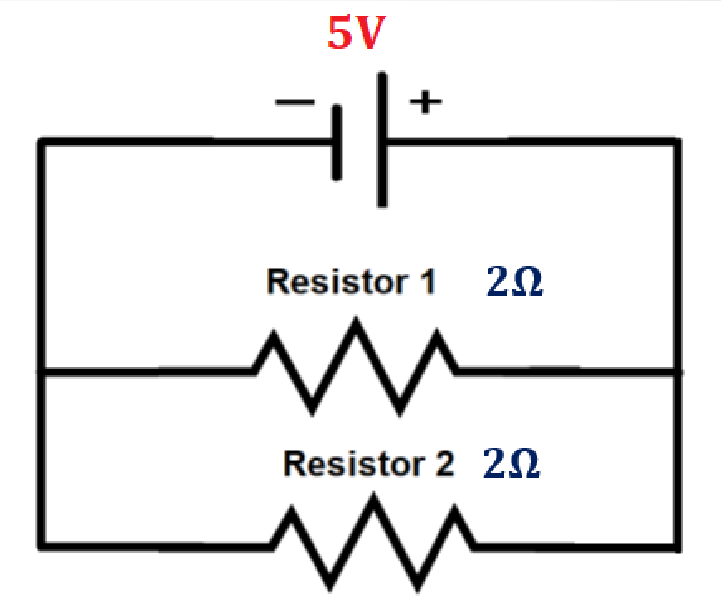
- In a parallel circuit the current “has a choice” of which path to take
- Some current will go down each path
- How much current takes each path is determined by the relative strengths of the resistors



The trivial case

- Consider a parallel circuit with only two resistors and where both resistors are the same strength
- In this case, the current will go equally down each path
- We know that the voltage drop on both resistors will be 5V, so using Ohm's Law

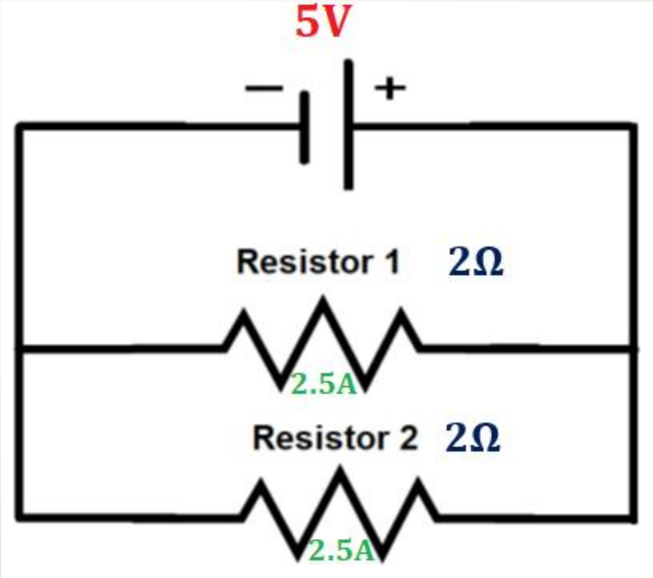
$$I_1 = \frac{V}{R_1} = \frac{5}{2} = 2.5A = I_2$$



So … what is the total current?

- Remember; current is a flow of electrons
- So … if 2.5A worth of electrons flows down one path and 2.5A worth of electrons flows down the other, then the total current will be …

$$2.5 + 2.5 = 5A$$



To generalise:

The total flow of current down parallel circuits will be the sum of each individual path

Formulas

- For resistance in series we have a lovely formula

$$R = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + \dots + R_n$$

- Is there a similar, Dirac quality, formula for parallel? Nah
  - But there is a formula

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \dots + \frac{1}{R_n}$$

Tackling big circuits

- With a few simple formulas … and a modicum of thought, you can tackle any size circuit.
- You just
  - “build the circuit up” and
  - “break the circuit down” as required